Tutorial Sheet-1: Propositional logic, Predicates, and Quantifiers

1. Which of these are propositions?
   (a) Do not pass. (b) What time is it? (c) $4 + x = 5$. (d) $2^n \geq 100$.

2. Consider the following propositions, $p :=$ Swimming at New Jersey shore is allowed, and $q :=$ Sharks have been spotted near the shore. Express each of these compound propositions as an English sentence.
   (a) $\neg q$ (b) $p \wedge q$ (c) $\neg p \vee q$ (d) $p \rightarrow \neg q$ (e) $p \leftrightarrow \neg q$ (f) $\neg p \wedge (p \vee \neg q)$

3. Let p, q, and r be propositions, $p :=$ Grizzly bears have been in the area, $q :=$ Hiking is safe on the trail, $r :=$ Berries are ripe along the trail. Write these propositions using p, q, and r and logical connectives.
   (a) Berries are ripe along the trail, but grizzly bears have not been seen in the area.
   (b) Grizzly bears have not been seen in the area and hiking on the trail is safe, but berries are ripe along the trail.
   (c) If berries are ripe along the trail, hiking is safe if and only if grizzly bears have not been seen in the area.
   (d) It is not safe to hike on the trail, but grizzly bears have not been seen in the area and the berries along the trail are ripe.
   (e) For hiking on the trail to be safe, it is necessary but not sufficient that berries not be ripe along the trail and for grizzly bears not to have been seen in the area.
   (f) Hiking is not safe on the trail whenever grizzly bears have been seen in the area and berries are ripe along the trail.

4. Determine whether each of these conditional statements is true or false.
   (a) If $1 + 1 = 2$, then $2 + 2 = 5$. (b) If $1 + 1 = 3$, then $2 + 2 = 4$
   (c) If $1 + 1 = 3$, then $2 + 2 = 5$ (d) If monkeys can fly, then $1 + 1 = 3$.

5. State the converse, contrapositive, and inverse of each of these conditional statements.
   (a) If it snows today, I will ski tomorrow.
   (b) I come to class whenever there is going to be a quiz.
   (c) A positive integer is a prime only if it has no divisors other than 1 and itself.

6. Construct a truth table for each of the following compound propositions:
   (a) $p \rightarrow \neg p$ (b) $p \leftrightarrow \neg p$ (c) $p \oplus (p \vee q)$ (d) $(p \wedge q) \rightarrow (p \vee q)$ (f) $(p \leftrightarrow q) \oplus (p \leftrightarrow \neg q)$

7. Show that the following statements are tautology by using and without using truth table.
   (a) $(p \wedge q) \rightarrow p$ (b) $p \rightarrow (p \vee q)$ (c) $\neg p \rightarrow (p \rightarrow q)$ (d) $\neg(p \rightarrow q) \rightarrow p$.

8. Prove or disprove the following.
   (a) $p \leftrightarrow q \equiv (p \wedge q) \vee (\neg p \wedge \neg q)$ (b) $\neg(p \leftrightarrow q) \equiv (p \leftrightarrow \neg q)$
   (c) $(p \leftrightarrow q) \wedge (p \rightarrow r) \equiv p \rightarrow (q \wedge r)$ (d) $(p \rightarrow r) \vee (q \rightarrow r) \equiv (p \wedge q) \rightarrow r$.

9. Check the validity of the following argument.
   $p \to \neg q$,
   $r \to q$,
   $\therefore r \to \neg p$.

10. Check the validity of the following argument.

    If you invest in the Gomermatic Corporation, then you get rich,

    You did not invest in the Gomermatic Corporation,

    Therefore you did not get rich.

11. Check the validity of the following argument. Given premises: $p \to q$, $\neg p \to r$, $r \to s$, and conclusion: $\neg q \to s$.

12. Let $P(x)$ be the statement "x spends more than five hours every weekday in class", where the domain for x consists of all students. Express each of these quantifications in English.
    $(a)\ \exists\, x\, P(x)$ $\quad$ $(b)\ \forall\, x\, P(x)$ $\quad$ $\exists\, x\, \neg P(x)$ $\quad$ $(d)\ \forall\ \neg P(x)$.

13. Suppose that the domain of the propositional function $P(x)$ consists of the integers 0, 1, 2, 3, and 4. Write down each of these propositions using disjunctions, conjunctions, and negations.
    $(a)\ \exists\, x\, P(x)$ $\quad$ $(b)\ \forall\, x\, P(x)$ $\quad$ $(c)\ \exists\, x\, \neg P(x)$ $\quad$ $(d)\ \forall\, x\, \neg P(x)$ $\quad$ $(e)\ \neg\, \exists\, x\, P(x)$.

14. Prove or disprove that $\exists x\ \forall y\ P(x, y) \not\equiv \forall y\ \exists x\ P(x, y)$.

15. Let $f : \mathbb{R} \to \mathbb{R}$ be a function and $a, b \in \mathbb{R}$. Write the negation of the following.
    (i) $\exists L \in \mathbb{R}\ \forall \epsilon > 0\ \exists \delta > 0\ \forall x \in \mathbb{R}\ \Big(0 < |x - a| < \delta \longleftrightarrow |f(x) - L| < \epsilon\Big)$, and
    (ii) $\forall \epsilon > 0\ \exists \delta > 0\ \exists \alpha > 0\ \forall x \in \mathbb{R}\ \exists y \in \mathbb{R}\ \Big((0 < |x - a| < \delta \text{ and } 0 < |y - b| < \alpha) \to (|f(x) - f(a)| < \epsilon \text{ or } |f(y) - f(b)| < \epsilon)\Big)$.

16. Express the negation of these propositions using quantifiers, and then express the negation in English.

    (a) Some drivers do not obey the speed limit.

    (b) All Swedish movies are serious.

    (c) No one can keep a secret.

    (d) There is someone in this class who does not have a good attitude.

17. Express the following specifications using predicates, quantifiers, and logical connectives.

    (a) Every user has access to an electronic mailbox.

    (b) The system mailbox can be accessed by everyone in the group if the file system is locked.

    (c) The firewall is in a diagnostic state only if the proxy server is in a diagnostic state.

    (d) At least one router is functioning normally if the throughput is between 100 kbps and 500 kbps and the proxy server is not in diagnostic mode.